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Fiber Beam Element Model for the Collapse Simulation of Concrete Structures under Fire

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Abstract In order to analyze and simulate the collapse of reinforced concrete (RC) elements under fire, a novel numerical model based on the fiber beam model is proposed in this paper. By dividing the cross section of beam element into many small concrete and steel fibers and assigning different materials to each fiber, this model can consider the non-uniform temperature distribution across the section and simulate the behavior of cracking or crushing for concrete and yielding for steel. The explicit tangential stiffness matrix is deduced for proposed fiber beam with Total Lagrangian (TL) description, and the incremental equilibrium equations are also established. Finally, the proposed fiber model is validated by comparing with various experimental results.

Key words: fiber model, beam element, fire, collapse, nonlinearity

INTRODUCTION

Fire is one of the most dangerous disasters that can destroy buildings. With the occurring of local damage or even complete collapse of buildings under fire, more and more attentions are paid to the security of structures under fire [1, 2]. As a result, many theoretical and experimental researches have been conducted on the behaviors of various structural elements, substructures and even whole structures under fire [3]. Traditional researches of structure under fire are based on single element test in the standard furnace. In the 1980's and the 1990's, performance based conception for fire design is adopted by BS5950 Part 8 and EC4[4] respectively in UK and Europe, which means the safety of a certain structure in fire can be determined with the fire resistant calculation. In China, the current codes including the fire safety code for building design and the code for fire protection design of tall buildings are still based on single element test in standard fire. With the improving requirements on disaster resistance of building structures, the performance-based fire resistant design method has drawn great attention [5], which requires the whole structural analysis for real fire situation. Due to the extreme large workload in the complex building simulations with solid finite elements, it is necessary to develop a suitable finite element model to study the failure process of the structural elements exposed to fire. Now, fiber beam model is widely used in reinforced concrete (RC) structures. Hence, a novel numerical model based on the fiber beam model is proposed in this paper to simulate the collapse of reinforced concrete beam and column under fire.

The model of fiber beam developed here is a two nodal element with six degrees of freedom at each node and the cross section of beam element is divided into many small concrete and steel fibers. The non-uniform stiffness of cross section along the beam, which is caused by the various kinds of loads and the degradation of materials at elevated temperatures, is considered with the 3-point Gauss integration along the axis of each fiber. Geometric nonlinearity of large displacement is also modeled with Total Lagrangian (TL) description, and the explicit tangential stiffness matrix is deduced for proposed fiber beam with the consideration of large displacements. By applying the incremental thermo elastic-plastic constitutive model, the incremental equilibrium equations are also established for the element of fiber beam. Finally, the analyzing results are compared with the data from fire experiments. The results show that the predictions of the model proposed in this paper agree well with the results of the tests and can be used to analyze and simulate the collapse of reinforced concrete elements under fire.

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THE ELEMENT MODEL OF FIBER BEAM

The fiber beam element consists of two nodes and six degrees of freedom at each node, as shown in Fig.1. The cross section of beam element is divided into many small concrete and steel fibers and the following assumptions are established: 1) Geometric nonlinearity due to large displacement is considered but the strain of material is still treated as small strain situation. 2) Plane-in-plane assumption is adopted to constraint the deformation of different fibers in the same section. 3) For each fiber only the longitudinal stress is considered. 4) The material of each fiber may be different, and the 3-point Gauss integration is considered along the axis of each fiber.3) Each fiber within the cross-section can have a different temperature, but this is uniform along the fiber.

The deformation of the fiber beam element is based on the Total Lagrangian (TL) method, and the displacements at any point on the reference axis between two end nodes can be expressed as follows:

$$\{u_0\} = [N]\{q\} \tag{1}$$

where $\{u_0\} = [u_0, \quad v_0, \quad w_0, \quad \theta_0]$ are the displacements at any point.

$[N]$ is the shape function matrix given by Bathe [6].

$\{q\}$ is the nodal displacement vector.

The displacements of any point A on any cross-section can be expressed as

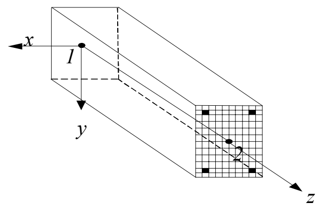

Fig.1: The fiber beam element

$$u = u_0 - y\sin\theta_y + x\sin\theta_x \tag{2-a}$$
$$v = v_0 - y + y\cos\theta_y \cos\theta_z + x\cos\theta_x \sin\theta_z \tag{2-b}$$
$$w = w_0 - x + x\cos\theta_x \cos\theta_z - y\cos\theta_y \sin\theta_z \tag{2-c}$$

where x, y are the coordinates of point A.

It can be seen from Fig.3 and Fig.4 that

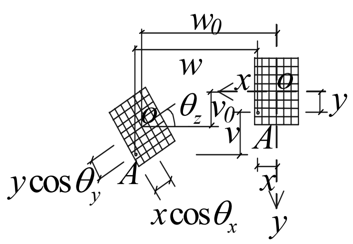

Fig.2: Element cross-section mesh

$$\sin\theta_x = \frac{dw_0}{dz} = w_0' \text{ and } \sin\theta_y = \frac{dv_0}{dz} = v_0' \tag{3-a}$$

then $\cos\theta_x = \sqrt{1-(w_0')^2}$ and $\cos\theta_y = \sqrt{1-(v_0')^2}$ (3-b)

if $\sin\theta_z = \theta_z$ and $\cos\theta_z = 1$ (3-c)

then the displacements of any point A can be expressed as

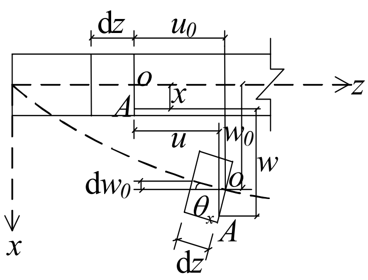

Fig.3: Deformations in plane xz

$$u = u_0 - yv_0' + xw_0' \tag{4-a}$$
$$v = v_0 - y + y\sqrt{1-(v_0')^2} + x\theta_z\sqrt{1-(w_0')^2} \tag{4-b}$$
$$w = w_0 - x + x\sqrt{1-(w_0')^2} - y\theta_z\sqrt{1-(v_0')^2} \tag{4-c}$$
$$\theta = \theta_0 \tag{4-d}$$

For each fiber only the longitudinal stress is considered, the stain of point A can be expressed as

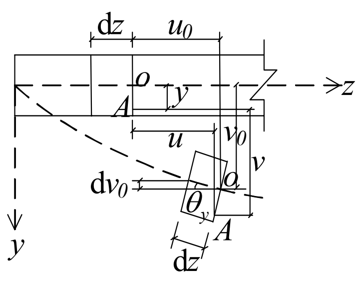

Fig.4: Deformations in plane yz

$$\varepsilon_z = \frac{\partial u}{\partial z} + \frac{1}{2}[(\frac{\partial u}{\partial z})^2 + (\frac{\partial v}{\partial z})^2 + (\frac{\partial w}{\partial z})^2] = \varepsilon_0 + \varepsilon_L \tag{5}$$

and $\varepsilon_0 = \frac{\partial u}{\partial z} = \langle B_0 \rangle \langle q \rangle$ (6-a)

$$\varepsilon_L = \frac{1}{2}[(\frac{\partial u}{\partial z})^2 + (\frac{\partial v}{\partial z})^2 + (\frac{\partial w}{\partial z})^2] \tag{6-b}$$

where ε_0 represents the small linear-displacement strains and ε_L represents the non-linear displacement stains.

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then $d\varepsilon_0 = \langle B_0\rangle\langle dq\rangle$ (7)

$$d\varepsilon_L = \langle q\rangle[\langle B_0\rangle^T\langle B_0\rangle + \langle B_1\rangle^T\langle B_1\rangle + \langle B_2\rangle^T\langle B_2\rangle]\langle dq\rangle = \langle q\rangle[B_L]\langle dq\rangle = \langle \overline{B}_L\rangle\langle dq\rangle \tag{8}$$

where $\langle B_0\rangle = \langle A \quad B'y \quad B'x \quad 0 \quad -C'x \quad C'y \quad -A \quad -B'y \quad -B'x \quad 0 \quad -D'x \quad D'y\rangle$,

$\langle B_1\rangle = \langle 0 \quad -B \quad 0 \quad 0 \quad 0 \quad C \quad 0 \quad B \quad 0 \quad 0 \quad 0 \quad -D\rangle$, $\langle B_2\rangle = \langle 0 \quad 0 \quad -B \quad 0 \quad -C \quad 0 \quad 0 \quad 0 \quad B \quad 0 \quad D \quad 0\rangle$,

$A = -1/l$, $B = 6z/l^2 - 6z^2/l^3$, $C = 3z^2/l^2 - 4z/l + 1$, $D = -3z^2/l^2 + 2z/l$, $E = 1 - z/l$, $F = z/l$

then $d\varepsilon_z$ can be expressed as

$$d\varepsilon_z = d\varepsilon_z + d\varepsilon_L = \langle B_0\rangle\langle dq\rangle + \langle \overline{B}_L\rangle\langle dq\rangle = \langle \overline{B}\rangle\langle dq\rangle \tag{9}$$

On the other hand, the total thermo elastic-plastic strain increment in each fiber can be expressed as

$$\{d\varepsilon\} = \{d\varepsilon_e\} + \{d\varepsilon_{e,T}\} + \{d\varepsilon_T\} + \{d\varepsilon_{cre}\} + \{d\varepsilon_P\} \tag{10}$$

where $\{d\varepsilon_e\}$=incremental elastic strain components, $\{d\varepsilon_{e,T}\}$=incremental strain components due to temperature dependent material properties, $\{d\varepsilon_T\}$=incremental thermal strain components, $\{d\varepsilon_P\}$= incremental plastic strain components.

When the isotropic hardening scheme is considered, the thermo elastic plastic constitutive equation can be expressed by [7]

$$\{d\sigma\} = [D_{ep}]\left(\{d\varepsilon\} - \{\alpha\}\, dT - \frac{\partial[D_e]^{-1}}{\partial T}\{\sigma\}\, dT - \{d\varepsilon_{cre}\} - \frac{[D_{ep}]^{-1}[D_e]\{\sigma'\}}{S}\frac{\partial F}{\partial T}\, dT\right) \tag{11}$$

where $[D_{ep}]$ is thermo elastic plastic constitutive matrix, α is the thermal expansion coefficient,

$S = \{\sigma'\}^T[D_e]\{\sigma'\} + H'$, $\{\sigma'\} = \left\{\frac{\partial F}{\partial \sigma}\right\}$, $[D_{ep}] = [D_e] - [D_p]$ and $[D_p] = [D_e]\{\sigma'\}\{\sigma'\}^T[D_e]/S$

Because only the longitudinal strain is considered for each fiber in the element, then

$[D_e] = E_0$, $[D_p] = 2G\sigma'^2/S_0$, $\sigma' = 2\sigma_z/3$, $S = 4\sigma_z^2(3G + H')/9$,

$S_0 = 2\sigma_z^2(1 + H'/3G)/3$, $H' = 1/(1/E_t - 1/E_0)$

where σ_z is the axial stress of the fiber at the gauss point, E_t is the tangential stiffness of the fiber at the gauss point

If $\overline{E} = E_0 - 2G\sigma'^2/S_0$, and $\frac{\partial[D_e]^{-1}}{\partial T} = -\frac{1}{E_0^2}\frac{\partial E_0}{\partial T}$, $\frac{[D_{ep}]^{-1}[D_e]\{\sigma'\}}{S} = \frac{2E_0\sigma_z}{3\overline{E}S}$ (12)

then $d\sigma_z = \overline{E}(d\varepsilon_z - d\varepsilon_z^T - d\varepsilon_{cre}) = \overline{E}\, d\varepsilon_z'$ (13)

where $d\varepsilon_z' = d\varepsilon_z - d\varepsilon_z^T - d\varepsilon_{cre}$ is the total incremental thermo mechanical strain, $d\varepsilon_z$ is the total incremental strain,

$d\varepsilon_z^T = \alpha\, dT - \frac{1}{E_0^2}\frac{\partial E_0}{\partial T}\sigma_z\, dT + \frac{2E_0\sigma_z}{3\overline{E}S}\frac{\partial F}{\partial T}\, dT$ is the incremental thermal strain.

Finally, the tangential stiffness matrix of the fiber beam can be deduced with the deformation equations and the thermo elastic plastic constitutive equation. The incremental strain energy in the element can be expressed as

$$\Delta U = 1/2\int_V \{d\varepsilon'\}^T\{d\sigma\}\, dv$$

$$= 1/2\{\Delta q\}^T\int_v \overline{\mathrm{B}}^T D\overline{\mathrm{B}} dv\{\Delta q\} + \int_v d\overline{\mathrm{B}}^T \sigma dv - \{\Delta q\}^T\int_v \overline{\mathrm{B}}^T D\{d\varepsilon^T\}dv - \{\Delta q\}^T\int_v \overline{\mathrm{B}}^T D\{d\varepsilon_{cre}\}dv + 1/2\int_v \{d\varepsilon^T\}^T D\{d\varepsilon^T\}dv \tag{14}$$

The work done on the element by the extern force is

$$\Delta V = -\langle dq \rangle^T \{dQ\} \tag{15}$$

So the total incremental potential energy can be expressed as

$$\prod_p = \Delta U + \Delta V \tag{16}$$

Upon substituting ΔU and ΔV into the expression for $\prod_p$ and applying the variation process leads to the following equation:

$$\frac{\partial \prod_p}{\partial \{dq\}} = \int_v \overline{\mathrm{B}}^T D \overline{\mathrm{B}} dv \, \{\Delta q\} - \int_v \overline{\mathrm{B}}^T D \{d\varepsilon^T\} dv - \int_v \overline{\mathrm{B}}^T D \{d\varepsilon_{cre}\} dv - \{dQ\} = 0 \tag{17}$$

It can be expressed as $K_e \{\Delta u\} = \{\Delta F\}$ (18)

where $K_e = K_0 + K_\sigma$ is the elastic-plastic stiffness matrix , K_σ is the geometry stiffness matrix and

$$K_0 = \int_v \overline{\mathrm{B}}^T D \overline{\mathrm{B}} dv = \int_v (\mathrm{B}_0^{\mathrm{T}} + \overline{\mathrm{B}}_{\mathrm{L}}^{\mathrm{T}}) D (\mathrm{B}_0 + \overline{\mathrm{B}}_{\mathrm{L}}) dv = \int_v \mathrm{B}_0^{\mathrm{T}} \overline{E} B_0 dv + \int_v \mathrm{B}_0^{\mathrm{T}} \overline{E \mathrm{B}}_{\mathrm{L}} dv + \int_v \overline{\mathrm{B}}_{\mathrm{L}}^{\mathrm{T}} \overline{E} B_0 dv + \int_v \overline{\mathrm{B}}_{\mathrm{L}}^{\mathrm{T}} \overline{E \mathrm{B}}_{\mathrm{L}} dv$$

$$K_\sigma = \int_v \mathrm{d}\overline{\mathrm{B}}^T \sigma dv = \int_v \mathrm{B}_{\mathrm{L}}^{\mathrm{T}} \sigma_z dv \, ,$$

$\{\Delta F\} = \Delta F_T + \Delta F_C + \Delta F_M$ is the incremental thermo mechanical load matrix.

$$\Delta F_T = \int_v (\mathrm{B}_0^T + \overline{B}_L^T) \overline{E} d\varepsilon_z^T dv = \int_v (\mathrm{B}_0^T + \overline{B}_L^T) \overline{E} (\alpha dT - \frac{1}{E_0^2} \frac{\partial E_0}{\partial T} \sigma_z dT + \frac{2}{3} \frac{E_0 \sigma_z}{\overline{E} S} \frac{\partial F}{\partial T} dT) dv$$

$$\Delta F_C = \int_v (\mathrm{B}_0^T + \overline{B}_L^T) \overline{E} d\varepsilon_c dv \text{ and } \Delta F_M = \{dQ\}$$

When the local equilibrium equation is transformed into global coordinates through the transformation matrix T, the structural equilibrium equations can be obtained.

$$T^T K_e T \{\Delta q\} = T^T \Delta F \tag{19}$$

and the incremental displacements can be obtained by solving these equations.

MATERIAL MODEL

Due to the extreme nonlinearity of concrete and steel under elevated temperature, the temperature dependent material models are required in order to perform numerical analysis of structure under fire.

Because only longitude deformation of each fiber is considered in the fiber beam model, the uniaxial material models are adopted. The models used here for beam is in accordance with EC4 part 1.2[4], which has previously been used by Huang [8] in concrete slabs analysis. the temperature-dependent stress-strain relationships for concrete in compression are shown in Fig.5. Concrete is assumed to have no residual strength in either compression or tension once it has crushed. The temperature-dependent stress-strain relationships for steel are shown in Fig.7. A bilinear model is adopt to present the tensile softening of cracked concrete (Fig.6 [9]), which can make the numerical solution process more stable.

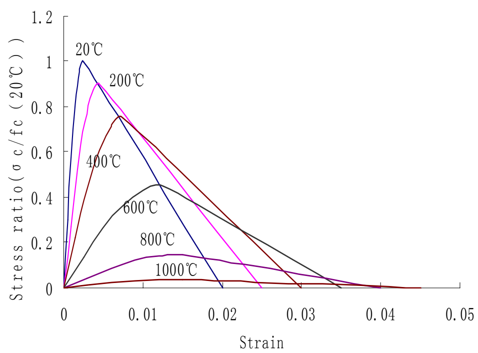

Fig.5: Stress-strain relationship of concrete at elevated temperatures

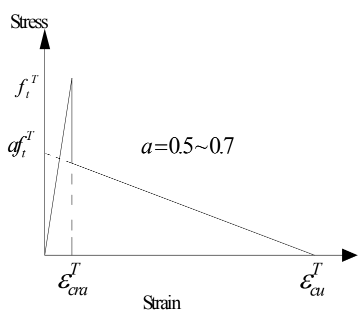

Fig.6: Stress-strain relationship of concrete in tension

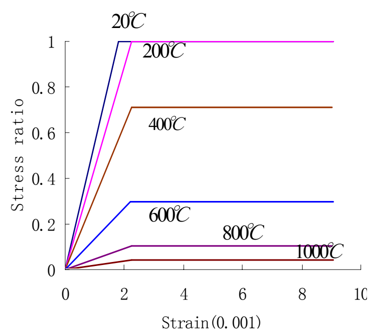

Fig.7: Stress-strain relationship of steel at elevated temperatures

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NUMERICAL EXAPMLE

Three representative fire tests of structural elements are analyzed to validate the fiber model proposed above. The first is a four-face heated column, which could prove the axial behavior of the section because of the symmetric temperature distribution. Then a three-face heated beam is used to testify the bend behavior of the beam section. Finally a three-face heated column subjected to a constant compressive force is analyzed, the column is bended because of the unsymmetrical temperature distribution and compressed because of the external axial pressure, so the coupled behaviors of bending and compression could be considered.

1 *Four-face heated column*

The fibers stress will change and stress redistribution will occur across the section because of the non-uniform temperature distribution. In order to testify the axial behavior of the section, the predictions of the proposed model are compared with the fire test results of a four-face heated reinforced concrete column with axial load, reported on by Lie and Irwin[10].the geometric of the column and steel reinforcement are shown in Fig.8. In the test, the pre-loaded axial force is 1067kN, and the fire situation is simulated by exposing the column to hot surrounding air and the temperature of the air changing follows the ASTM fire curve. The yield stress of the reinforcement is 414MPa, the compressive strengthen of concrete is 36.1MPa and the measured fire resistance time of the column is 208 min [10].

The temperature distribution on the cross-section of the column should be determined before the structural analysis; the transient heat conduction problem is solved by the computer program developed by the authors [11], which has been validated by comparing the predictions with the tests results. The material parameters needed in the analysis, including the heat conductivity and specific heat, are estimated according to Eurocode 4[4]. The other parameters such as heat transfer coefficient and heat emissivity are 25W/mK and 0.2 respectively. Fig.10 shows the calculated and measured temperature distributions in concrete cross-section at various times.

In the structural analysis, the column is simulated with one beam element, the cross-section is divided into fifty concrete fibers and four steel fibers and each fiber has three Gauss points, as a result there are 162 Gauss points together in one beam element. Fig.10 shows the calculated and measured axial displacements at various times. From the results of the test, it can be found that the axial displacement increases during the first 120 min, showing that the column is elongated due to the growing thermal strain. Subsequently, with the increasing of the temperature, the axial displacement starts to decrease due to the degradation of the materials and becomes negative at 180 min. the column collapses at 208 min. From the prediction of the model, it can be found that the largest displacement occurs at 105 min. It is very close to the measured value 120 min. Finally, the displacement increases rapidly at about 215 min which shows that the column collapses under fire. Though the prediction of the formula proposed by Lie[10] was 200min, the numerical model proposed here are more flexible than the empirical formula by Lie[10] .

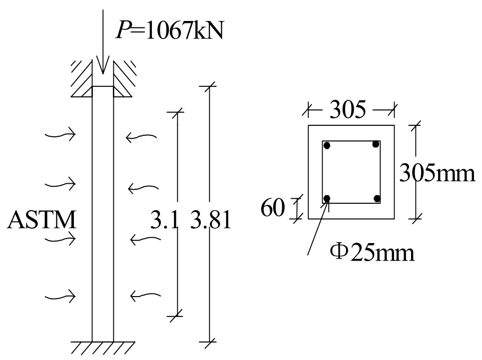

Fig.8: Four-face heated column in fire

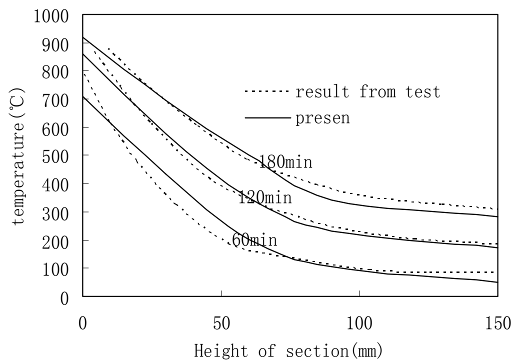

Fig.9: Temperature from test and calculation

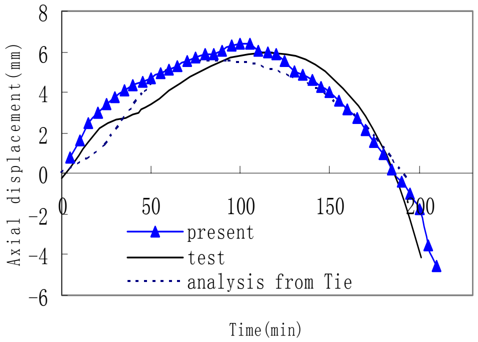

Fig.10: The development of axial displacement of the column

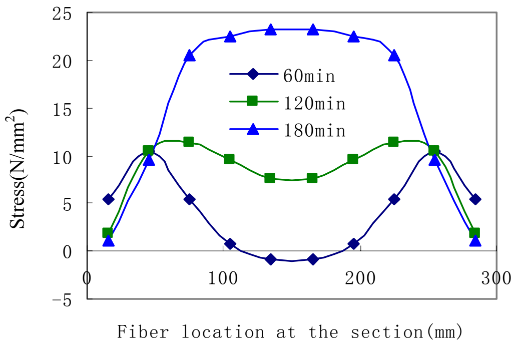

Fig.11: The develpment of the section stress

2 *Three-face heated beam*

In this example, the proposed model is testified by comparing the its predictions with the fire test results of a three-face heated four-point bended RC beam [14]. The dimension of the column and the loading data are given in Fig.12. The yield stress of the reinforcement is 270MPa; the compressive strength of concrete is 28.9MPa. The concentrated load P_0 is 4kN, the temperature of the air in the furnace measured in the test [14] is used to simulate the fire situation in the transient thermal analysis.

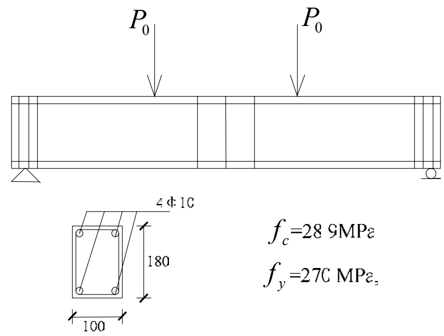

Fig.12:Three-face heated beam

In the structural analysis, in order to obtain the displacement at the mid-point of the column conveniently, the column is meshed into four beam elements. The cross-section is divided into fifty concrete fibers and four steel fibers and each fiber has three Gauss points, as a result there are 162 Gauss points together in one beam element. Fig.13 shows the calculated and measured displacements of the mid-span of the beam at various times.

From the results of the test, It can be seen that the mid-span displacement increases slowly when the temperature is below 400℃. After that, its deformation accelerates due to the thermal strain and non-uniform temperature distribution. When the temperature exceeds 550℃, though the level of non-uniform temperature reduces, the mid-span displacement increases rapidly and the beam collapses because the materials degenerate. From the calculation results, It can be seen that the predictions of the model agree well with the test results when the temperature is low. When the temperature exceeds 400℃, the predictions became some smaller than the test results because the temperature effect of cracking on the reinforcement is not considered in the numerical model. But the mid-span displacement increases rapidly and the beam collapses after 550℃, which agrees well with the test results.

The model is applied further to the analysis of the same beam subjected to different levels of the external loads and Fig.14 shows the numerical results. When the load level is very low and P_0 equals 2kN, the mid-span displacement increases slowly until the temperature reaches 1000℃.With the increase of the load level, the ultimate temperature of the beam reduces to 600℃ When P_0 equals 4kN. After that, the ultimate temperature of the beam still reduced but the speed of reduction became much slower. These predictions agree well with the test results, so the proposed model can be used to simulate the collapse of reinforced concrete beams under fire.

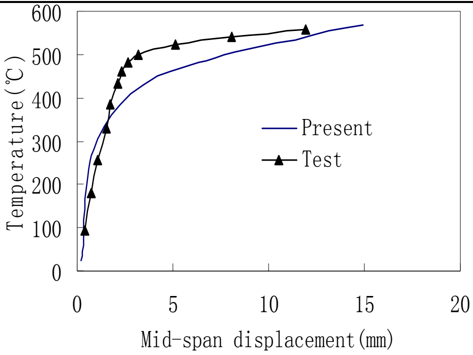

Fig.13: Development of the mid-span displacements

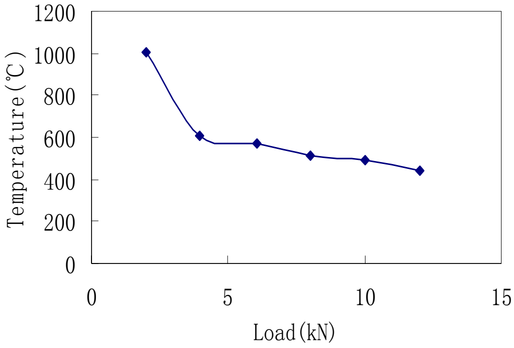

Fig.14: Relationship of load and ultimate temperature

3 *Three-face heated column*

In this example, the predictions of the proposed model are compared with the fire test results of a three-face heated RC column [14]. The geometric of the column and the loading data are given in Fig.15. The yield stress of the reinforcement is 414MPa; the compressive strength of concrete is 36.1MPa. In the test, the pre-loaded force is 180kN; the measured temperature of the air in the furnace is used to simulate the fire situation in the transient thermal analysis.

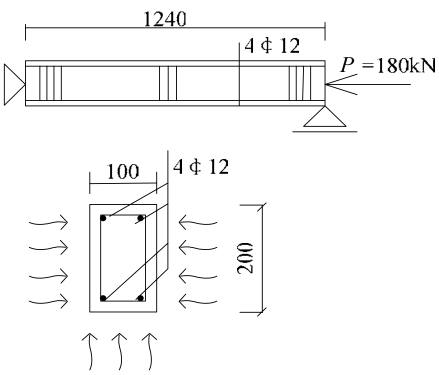

Fig.15: Three-face heated column

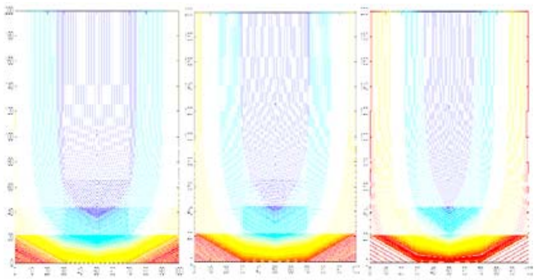

Fig.16: Temperature-distribution at 30min 60min and 120min

The temperature distribution on the cross-section of the column is also calculated with the programme developed by the authors. Fig.16 shows the results of the variation of temperature-distribution at 30min, 60min and 120min respectively. In the structural analysis, in order to obtain the displacement at the mid-point of the column conveniently, the column is meshed into two beam elements. The cross-section is also divided into fifty concrete fibers and four steel fibers and each fiber has three Gauss points, as a result there are also 162 Gauss points together in the beam element. Fig.17 shows the calculated and measured axial displacements at various times. Fig.18 shows the calculated and measured lateral displacements at various times. From the results of the test, It can seen that the axial displacement increases due to the thermal strain when the temperature is below 700℃. After that, the materials degenerate in high temperature. the lateral displacement was positive during the first 100 min, showing that the column bends to the heated side of the column. Subsequently, the column bends to the other side of the column due to the degradation of materials with the increasing of the temperature and the lateral displacement became negative and the column collapsed at 150 min.From the prediction of the model, It can seen that the largest axial displacement occurs at about 620℃ and increases rapidly at about 900℃ when the column collapses. At the same time, the lateral displacement also increases rapidly at about 140 min, which shows that the predicted fire resistance of the column is about 140 min.

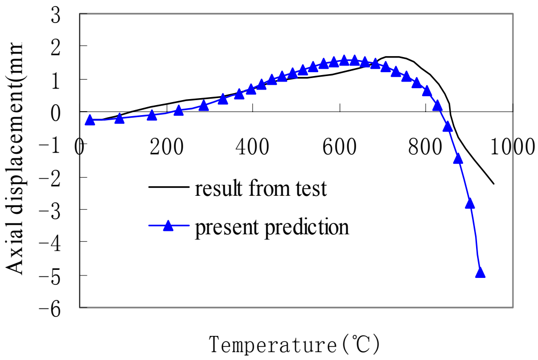

Fig.17:The variation of axial displacement

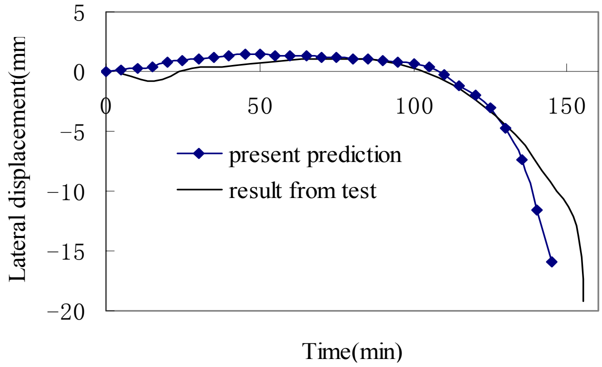

Fig.18:The variation of lateral displacement of the column

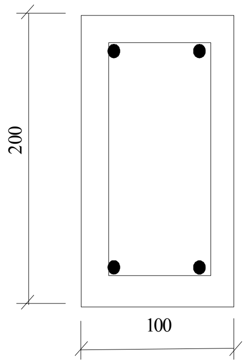

Fig.19:Cross-section of the column

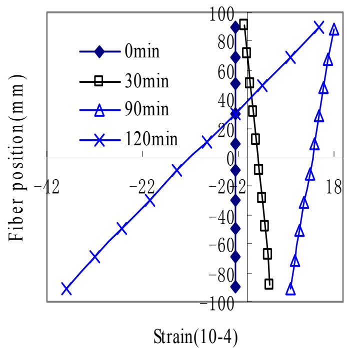

Fig.20:Variation of the section strain

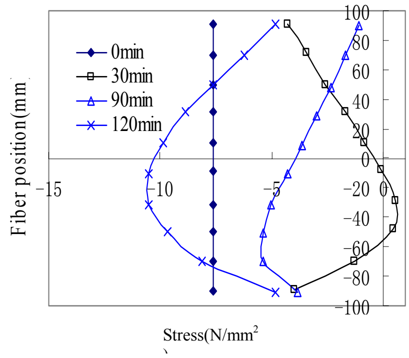

Fig.21:Variation of the section stress

CONCLUSIONS

A fiber beam model was developed in this paper. By applying the incremental thermo elastic plastic constitutive law, the proposed model was used to analyze the collapse of reinforced concrete structural elements under fire. The model considered the non-uniform temperature distribution across the section and nonlinear behavior of the materials.

Three representative element fire tests were analyzed to validate the fiber model proposed in this paper, and the predictions of this model agree well with the test results, it indicates that the fiber beam proposed here is capable of analyzing and simulating the collapse of reinforced concrete elements under fire with acceptable workload.

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