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# ELASTIC-PLASTIC ANALYSIS OF RC SHEAR WALL USING DISCRETE ELEMENT METHOD

Lu Xinzheng, Jiang Jianjing

Department of Civil Engineering, Tsinghua University, Beijing, 100084

**Abstract**: An analytic method of RC structure using discrete element method is introduced in this paper. The RC structures are meshed with concrete discrete elements and re-bar elements. The discrete elements are connected with "point to point" contact elements and spring elements. The damage of concrete is assumed that it only happens on the interfaces of different discrete elements. Hence, the contact estimation of traditional discrete element method is simplified and the stability and speed of calculation process is improved. The influence of crack surfaces also can be obtained in this method, which is difficult for normal finite element method. A two-limb shear wall model is analyzed using this method. The results show this method is rational and effective.

**Key Words**: Discrete Element, Shear Wall, Elastic-plastic Analysis

## 1. Introduction

In the traditional finite element analysis, the material is assumed to be homogeneous and continuous. However, if the structure continuum or cracks and crushes happening, it is difficult to analyze using finite element method. The discrete element method can effectively describe the discontinuity of material. But the contact estimation is very complex and often leads the calculation process to be unstable. In the practical RC structures, the width of cracks is very small comparing with the size of structures. It implies us to use an improved discrete element method to simplify the contact estimation while keeping its character.

## 2. Basic Assumption

(1). The concrete is meshed with concrete discrete elements and interface elements.
(2). The deformation of structure happens mainly in the discrete elements, while the damage only happens in the interface elements.
(3). The width of cracks is very small comparing with the size of structure. So the "point to point" contact estimation on the corner points can determine the relative condition of neighborhood elements.

## 3. Details of Elements

### *3.1. Concrete Elements*

As shown in Figure 1, polygon (1) and (2) are two concrete discrete elements. Lines AB and CD are the common edge of them. The interface element (3) is inserted into the public edge. The displacements of interface element corner points A', B', C', D' are consistent with the corner points A, B, C, D respectively.

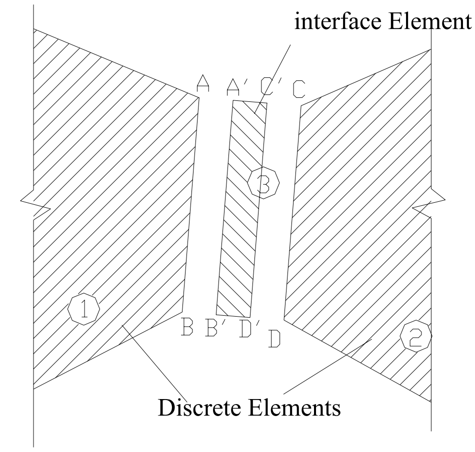

Figure 1. Discrete Elements and Interface Element

The details of interface element are shown as Figure 2. It is composed by two groups of combination elements that connect the corner points A', C' and B', D' respectively. There is a "point to point" contact element and two spring elements in each group, which are named as $C_p$, $C_t$ and $C_s$. They resist the press force, tension force and shear force respectively, which are caused by the relative displacement between the two corner points. Let $u$ , $v$ present the relative displacement of the two corner points in the local coordination. Then the element matrixes of these combination elements are discussed as following:

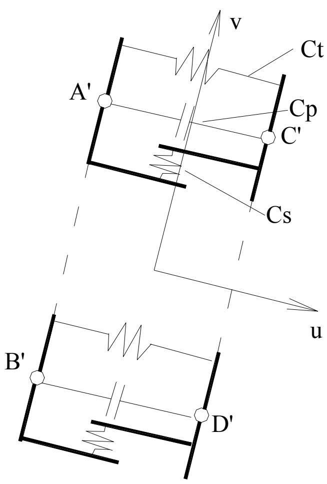

Figure 2. Details of Interface Element

3.1.1 Contact element $C_p$

If $u_{A'} > u_{C'}$, then

$$K_{Cp}^e = \begin{bmatrix} k_n & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & k_n & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{1}$$

If $u_{A'} < u_{C'}$, then

$$K_{Cp}^e = 0 \tag{2}$$

If $F_{Cp} > \dfrac{tlf_c}{2}$, which implies that the concrete is crushed, then the normal stiffness

$$k_n = 0\,. \tag{3}$$

Here $t$ is the thickness of the concrete, $l$ is the length of the comon edge and $f_c$ is the compression strength of concrete.

3.1.2 Spring element $C_t$

If $F_{Ct} \le \dfrac{tlf_t}{2}$, then

$$K_{Ct}^e = \begin{bmatrix} k_n & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & k_n & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \tag{4}$$

Here $f_t$ is the tension strength of concrete.

If $F_{Ct} > \dfrac{tlf_t}{2}$, which implies that the concrete is cracked, then

$$K_{Ct}^e = 0 \tag{5}$$

3.1.3 Spring element $C_s$

If $F_{Cs} \le \frac{tlf_v}{2}$, then

$$K_{Cs}^e = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & k_v & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k_v \end{bmatrix}, \tag{6}$$

Here $f_v$ is the shear strength of concrete.

If $F_{Cs} > \frac{tlf_v}{2}$, then

$$k_v = 0 \tag{7}$$

Because it is assumed that the deformation of concrete is mainly in the discrete elements, the stiffness coefficients $k_n$, $k_v$ in the discussion above can be set as a very large value, which implies that the deformation in the interface element is very small if there is no failure happens.

Hence, the material mechanics characters of concrete can be reflected by adjust the interface elements in this method. As the contact problem of discrete element is assumed to be "point to point" contact, the contact estimation is simplified. In the practical analysis, the combination elements have very small survival stiffness after they fail. So the break of calculation process due to too many failure of elements can be removed. The whole problem can be solved by displacement method, while the traditional discrete element using force method, which often cause instability.

As most of the mechanics characters of concrete are reflected in the interface elements, and the damage happens only in the interface, we just need to let the concrete discrete element to be normal non-linear elastic material.

***3.2. Re-bar Element***

In this method, the re-bars are treated in two ways.

3.2.1. Case I. Re-bars are in the concrete discrete element, as shown in Figure 3.

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Here the smeared reinforcement model (Jiang, 1994) is used to deal with the concrete and the re-bars.

$$\left[D_{s}\right]=E_{s}\begin{bmatrix}\rho_x & & & & & \\ & \rho_y & & & & \\ & & \rho_z & & & \\ & & & 0 & & \\ & & & & 0 & \\ & & & & & 0\end{bmatrix} \quad (8)$$

Here $E_s$ is the elastic module of re-bar. $\rho_x$, $\rho_y, \rho_z$ are the reinforcement ratio of direction x, y, z.

$$[D]=[D_c]+[D_s] \quad (9)$$

The element stiffness matrix is

$$[K]=\int B^T[D]Bdv \quad (10)$$

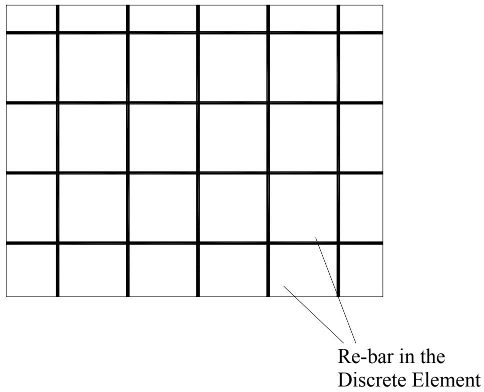


Figure 3. Re-bar in the discrete element

3.2.2. Case II. Re-bar is across the interface, as shown in Figure 4.

Here the effect of re-bar is assigned to the two corner points. Spring elements $S_1$ and $S_2$ are used to instead the re-bar. The stiffness $K_{s_1}=\frac{E_s Ab}{l}$, $K_{s_2}=\frac{E_s Aa}{l}$. So the force of the two spring elements is

$$\begin{Bmatrix}F_{x,s1}\\ F_{y,s1}\end{Bmatrix}=K_{s1}\begin{Bmatrix}\Delta x_{C'}-\Delta x_{A'}\\ \Delta y_{C'}-\Delta y_{A'}\end{Bmatrix} \quad (11)$$

$$\begin{Bmatrix}F_{x,s2}\\ F_{y,s2}\end{Bmatrix}=K_{s2}\begin{Bmatrix}\Delta x_{D'}-\Delta x_{B'}\\ \Delta y_{D'}-\Delta y_{B'}\end{Bmatrix} \quad (12)$$

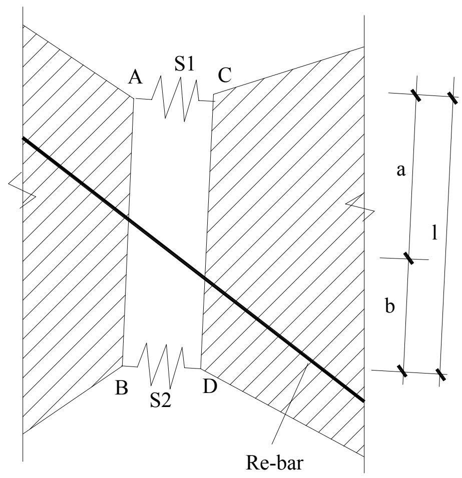


Figure 4. Re-bar across the interface

Here $\Delta x, \Delta y$ is the displacement of corner points along the axis x and y. If the axial force of re-bar larger than the yield strength, The tangent elastic module of re-bar is set to zero.

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## 4. Example

In order to verify the method presented above, a group of two-limb shear wall models tested by Fang and Li (1981) are analyzed. The shape, dimension and reinforcement of the model are show as Figure 5. The mechanics character of steel wires is shown in table 1. The strength of concrete is C25.

Table 1. The mechanics character of steel wires

| Type | Diameter （cm） | Area ($cm^2$) | Yield strength (MPa) | Ultimate strength (MPa) | Elastic sodule (GPa) |
|---|---|---|---|---|---|
| 8# | 0.40 | 0.1257 | 305 | 439 | 197 |
| 12# | 0.278 | 0.0607 | 298 | 432 | 214 |

Figure 6 shows the mesh of the shear wall. Figure 7 shows the load-displacement curve of model under one-way load. The figure shows that the numerical result is wall consisted with the test result. Figure 8. shows the load-displacement hysteresis curve of numerical results. It should be emphasized that the curves between points A, B, and C, D, which shows influence of the close and re-contact of crack surface. Because of the damage accumulation, the curve between points C and D is longer than that of points A and B, which implies that there are more cracks happened and the cracks are wider after a load-cycle.

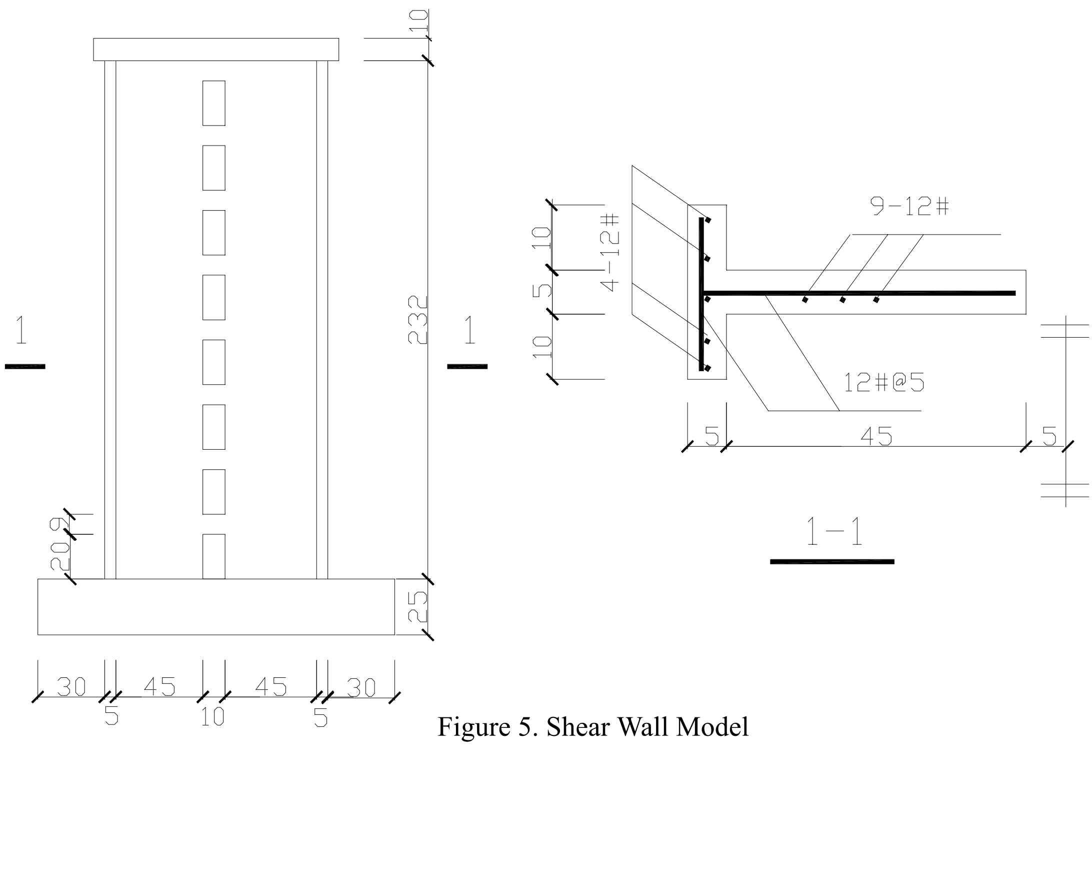


Figure 5. Shear Wall Model

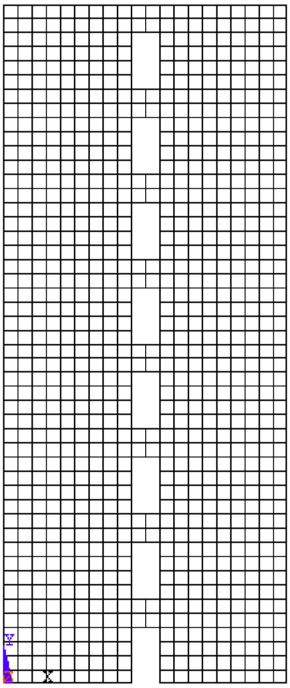


Figure 6. Mesh of model

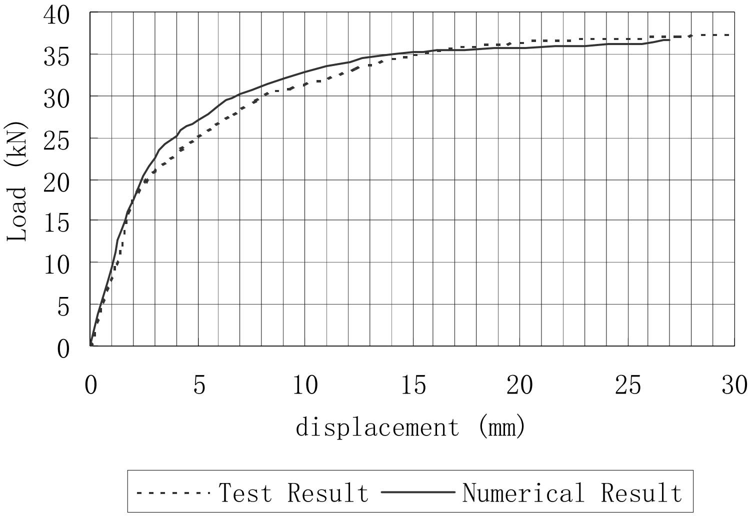


Figure 7. Load-displacement curve under one-way load

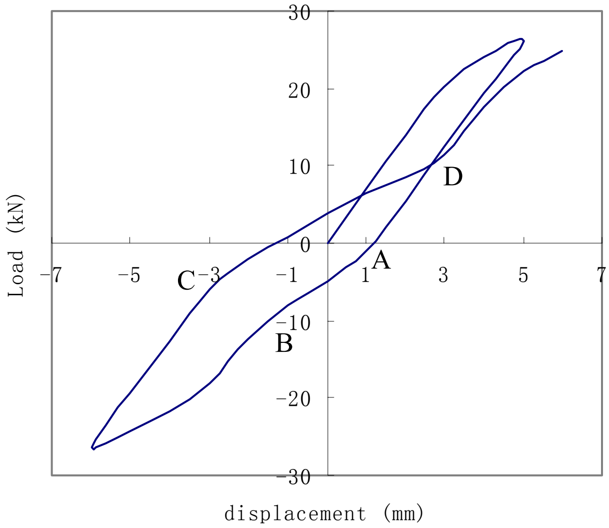


Figure 8. Load-displacement hysteresis curve

Figure 9 show the cracks and displacements of the shear wall. The details of discrete elements

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and interface elements are shown in Figure 10, which implies that the results of calculation are consisted with the real conditions.

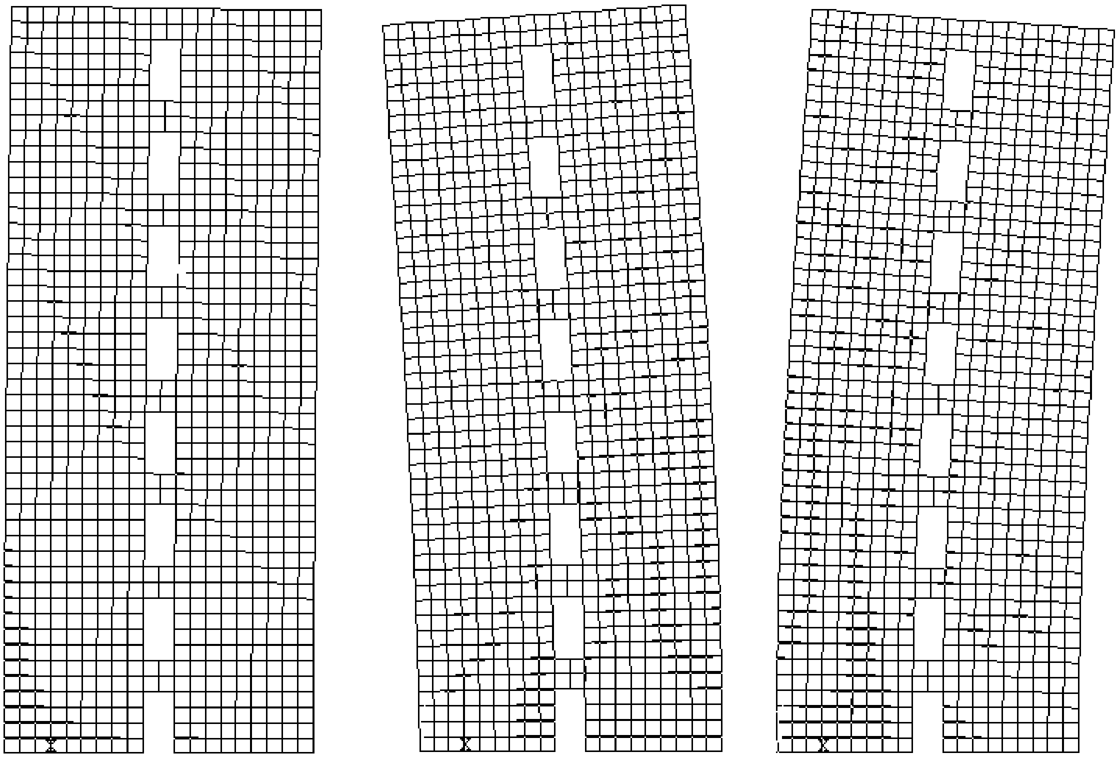

Figure 9. Cracks and displacements of the shear wall

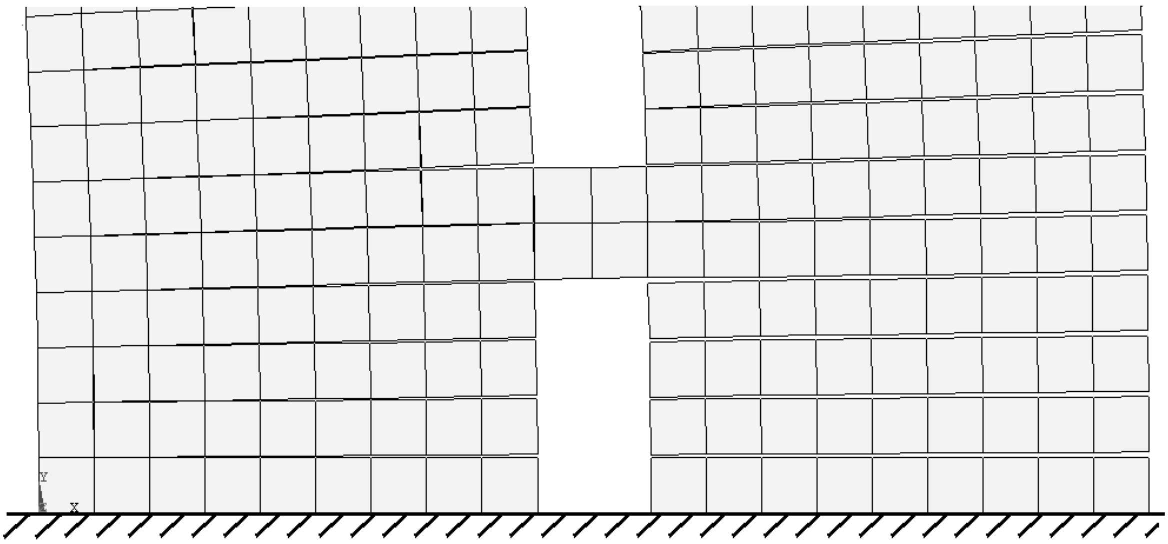


Figure 10. Details of discrete elements and Interface elements

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## 5. Dynamic Analysis

An Initial acceleration, whose value is 10 m/s$^2$, is applied to the base of the shear wall model. The variety of the top story displacement to time is shown in Figure 11. It can be obtained that when the structure damaged, the stiffness is decreased and the vibration period is longer.

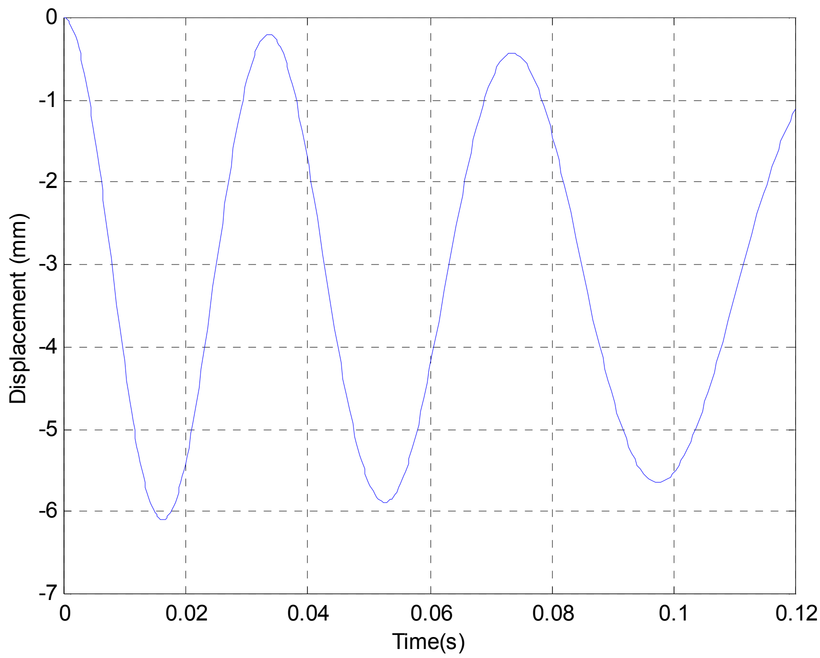


Figure 11. Dynamic response

## 6. Discussion and Conclusion

An analytic method of RC shear wall using discrete element method is introduced in this paper. A two-limb shear wall model is analyzed with this method. The numerical result shows that it is consisted with the test results. The load-displacement hysteresis curve can be obtained with this method while the cracks and deformation of structure can be clearly displayed. The dynamic results also show the influence of damage to the vibration period. There are more than 13000 combination elements are used in this case, which implies that powerful computer is needed in using this method.

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